Mathematical Calculations Progression Policy

UN Convention on the Rights of the Child	
Article 28	You have the right to a good quality education. You should be encouraged to go to school to the highest level you can

Rationale

The National Numeracy Framework provides a structured and systematic approach to teaching number. There is a considerable emphasis on teaching mental calculation strategies. Up to the age of 9 (Year 4) informal written recording should take place regularly and is an important part of learning and understanding. **More formal written methods** should follow **only** when the child is able to use a wide range of mental calculation strategies

Aims and objectives

This policy aims to standardise which strategies will be taught in which year group and how children will record their calculations.

Principles of teaching and learning

When are children ready for written calculations?

Addition and subtraction

- Do they know addition and subtraction facts to 20?
- Do they understand place value and can they partition numbers?
- Can they add three single digit numbers mentally?
- Can they add and subtract any pair of two digit numbers mentally?
- Can they explain their mental strategies orally and record them using informal jottings?

Multiplication and division

- Do they know the 2, 3, 4, 5 and 10 time table
- Do they know the result of multiplying by 0 and 1?
- Do they understand 0 as a placeholder?
- Can they multiply two and three digit numbers by 10 and 100?
- Can they double and halve two digit numbers mentally?
- Can they use multiplication facts they know to derive mentally other multiplication facts that they do not know?
- Can they explain their mental strategies orally and record them using informal jottings?

The above lists are not exhaustive but are a guide for the teacher to judge when a child is ready to move from informal to formal methods of calculation.

Differentiation for SEN and Able Children
In many classes, children will be at different stages in their move towards efficiency. This process should not be rushed; children should be moved on when they are ready.

Monitoring and assessment

It is important that procedures are in place to ensure that all staff are aware of the progression through calculations, and that children are being taught appropriate methods for their age and ability, which are in line with the agreed policy.

Attached to this policy is a summary document which gives examples of the progression in calculations for all operations. The Maths Manager will ensure that this is followed by all staff.

Review and Monitoring

The policy will be monitored and evaluated by:

- The Maths manager.
- Senior Management Team (SMT).
- LA Advisors & Inspectors.
- Governors.

The policy and/or procedures will, where necessary, be revised in light of these evaluations.

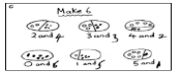
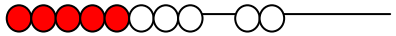
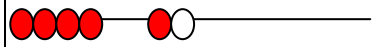
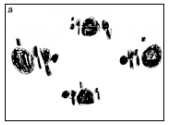
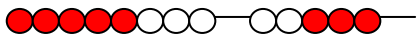
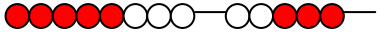
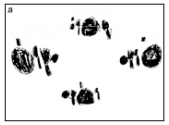
This policy should be read in conjunction with all other school policies in particular Special Educational Needs, Equal Opportunities, Able Child, Teaching and Learning and Assessment.

The Governing Body believes that fairness and consistency of judgement is essential to the operation of the school. All members of the school have equality of opportunity to achieve their full potential and will not be discriminated against because of age, disability, gender, sexual orientation, nationality, race, or religion. The Governing Body believes that the school always has to be aware of the potential for unconscious discrimination, to avoid assumptions about individual members of the school based on stereotypes and to use the teaching and learning arrangements actively to encourage everyone to achieve their full potential. All our policies are consistent with our duty of care to protect our pupils and to provide a learning environment that is safe and healthy. In all our dealings, we respect the strict code of confidentiality that underpins our school ethos.

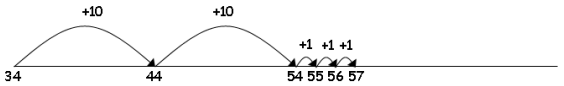
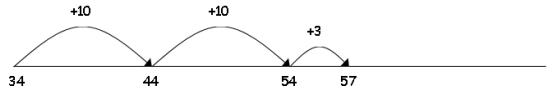
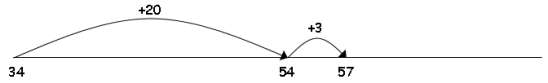
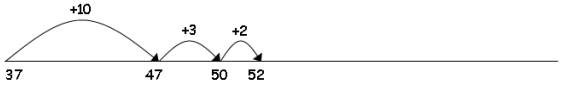
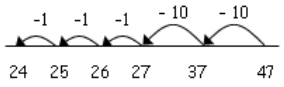
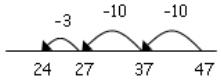
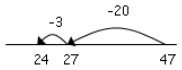
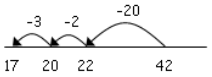
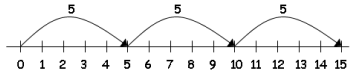
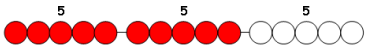
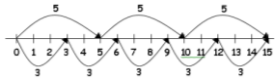
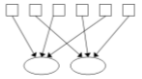
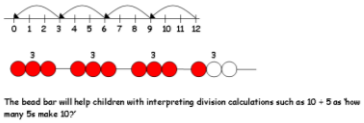
Policy date: October 2010

Review date: October 2014

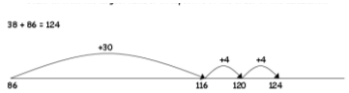
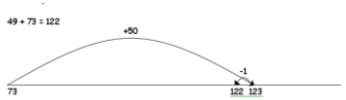
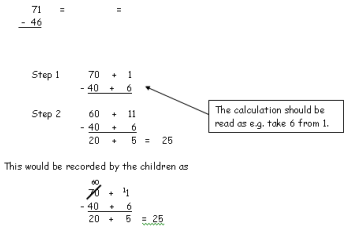
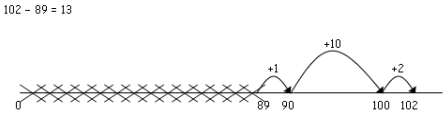
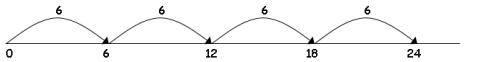
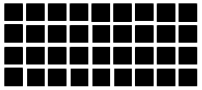
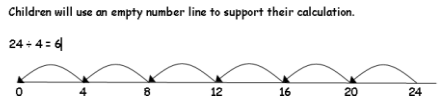
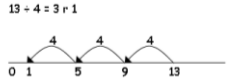
Mathematical Calculations Progression

	Addition	Subtraction	Multiplication	Division
YR	Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures, etc.  Bead strings or bead bars can be used to illustrate addition  8+2=10 They use numberlines and practical resources to support calculation and teachers *demonstrate* the use of the numberline.	Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They develop ways of recording calculations using pictures etc.  Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.  6-2= They use numberlines and practical resources to support calculation. Teachers *demonstrate* the use of the numberline.	Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups. 	Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s. 
Y1	using pictures  Bead strings or bead bars can be used to illustrate addition including bridging through ten by counting on 2 then counting on 3.  They use numberlines and practical resources to support calculation and teachers *demonstrate* the use of the numberline. Children then begin to use numbered lines to support their own calculations using a numbered line to count on in ones.	using pictures  Bead strings or bead bars can be used to illustrate subtraction including bridging through ten by counting back 3 then counting back 2.  13-5= Children then begin to use numbered lines to support their own calculations - using a numbered line to count back in ones. The numberline should also be used to show that 6 - 3 means the 'difference between 6 and 3' or 'the difference between 3 and 6' and how many jumps they are apart.	Children will experience equal groups of objects. They will count in 2s and 10s and begin to count in 5s. They will work on practical problem solving activities involving equal sets or groups. 	Children will understand equal groups and share items out in play and problem solving. They will count in 2s and 10s and later in 5s. 

Mathematical Calculations Progression

	Addition	Subtraction	Multiplication	Division
Y2	Children will begin to use 'empty number lines' themselves starting with the larger number and counting on. ✓ First counting on in tens and ones. 34 + 23 = 57  ✓ Then helping children to become more efficient by adding the units in one jump (by using the known fact 4 + 3 = 7). 34 + 23 = 57  ✓ Followed by adding the tens in one jump and the units in one jump. 34 + 23 = 57  ✓ Bridging through ten can help children become more efficient. 37 + 15 = 52  	Children will begin to use empty number lines to support calculations. **Counting back:** ✓ First counting back in tens and ones. 47 – 23 = 24  ✓ Then helping children to become more efficient by subtracting the units in one jump (by using the known fact 7 – 3 = 4). 47 – 23 = 24  ✓ Subtracting the tens in one jump and the units in one jump. 47 – 23 = 24  ✓ Bridging through ten can help children become more efficient. 42 – 25 = 17  Counting on: The number line should still show 0 so children can cross out the section from 0 to the smallest number. They then associate this method with 'taking away'.	Children will develop their understanding of multiplication and use jottings to support calculation: ✓ **Repeated addition** 3 times 5 is 5 + 5 + 5 = 15 or 3 lots of 5 or 5 x3 Repeated addition can be shown easily on a number line: 5 x 3 = 5 + 5 + 5  and on a bead bar: 5 x 3 = 5 + 5 + 5  ✓ **Commutativity** Children should know that 3 x 5 has the same answer as 5 x 3. This can also be shown on the number line.  ✓ **Arrays** Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.  	Children will develop their understanding of division and use jottings to support calculation ✓ **Sharing equally** 6 sweets shared between 2 people, how many do they each get?  ✓ **Grouping or repeated subtraction** There are 6 sweets, how many people can have 2 sweets each?  ✓ **Repeated subtraction using a number line or bead bar** 12 ÷ 3 = 4  ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations** □ ÷ 2 = 4 20 ÷ △ = 4 □ ÷ △ = 4

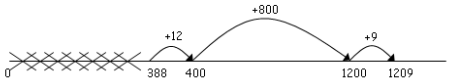
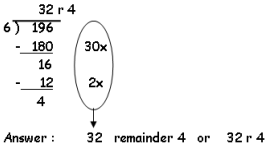
Mathematical Calculations Progression

	Addition	Subtraction	Multiplication	Division
Y3	Children will continue to use empty number lines with increasingly large numbers, including compensation where appropriate. ✓ Count on from the largest number irrespective of the order of the calculation.  ✓ Compensation  Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies. Adding the least significant digits first 67 + 24 11 (7 + 4) 80 (60 + 20) 91 267 + 85 12 (7 + 5) 140 (60 + 80) 200 352	Children will continue to use empty number lines with increasingly large numbers. Children will begin to use informal pencil and paper methods (jottings). ✓ **Partitioning and decomposition** • Partitioning – demonstrated using arrow cards • Decomposition - base 10 materials **NOTE** When solving the calculation 89 – 57, children should know that 57 **does NOT EXIST AS AN AMOUNT** it is what you are subtracting from the other number. Therefore, when using base 10 materials, children would need to count out only the 89. 89 = 80 + 9 - 57 50 + 7 30 + 2 = 32 ✓ **Begin to exchange.**  Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used.  	Children will continue to use: ✓ **Repeated addition** 4 times 6 is 6 + 6 + 6 + 6 = 24 or 4 lots of 6 or 6 x 4 Children should use number lines or bead bars to support their understanding.   ✓ **Arrays** Children should be able to model a multiplication calculation using an array. This knowledge will support with the development of the grid method.  9 x 4 = 36 9 x 4 = 36 ✓ **Scaling** e.g. Find a ribbon that is 4 times as long as the blue ribbon  ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations** □ x 5 = 20 3 x △ = 18 □ x ○ = 32 ✓ **Partitioning** 38 x 5 = (30 x 5) + (8 x 5) = 150 + 40 = 190	Ensure that the emphasis in Y3 is on grouping rather than sharing. Children will continue to use: ✓ **Repeated subtraction using a number line**  Children should also move onto calculations involving remainders.  ✓ **Using symbols to stand for unknown numbers to complete equations using inverse operations** 26 ÷ 2 = □ 24 ÷ △ = 12 □ ÷ 10 = 8

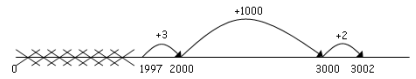
Mathematical Calculations Progression

	Addition	Subtraction	Multiplication	Division
Y4	✓ Carry below the line. 625 + 48 = 673 (1) 783 + 42 = 825 (1) 367 + 85 = 452 (11) *Using similar methods, children will:* ✓ *add several numbers with different numbers of digits;* ✓ *begin to add two or more three-digit sums of money, with or without adjustment from the pence to the pounds;* ✓ *know that the decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. £3.59 + 78p.*	✓ **Partitioning and decomposition** 754 − 86 = Step 1: 700 + 50 + 4 − 80 + 6 Step 2: 700 + 40 + 14 − 80 + 6 *(adjust from T to U)* Step 3: 600 + 140 + 14 − 80 + 6 = 600 + 60 + 8 = 668 *(adjust from H to T)* This would be recorded by the children as: 600 140 754 − 80 + 6 = 600 + 60 + 8 = 668 ✓ **Decomposition** 614 1 754 − 86 = 668 *Children should:* ✓ *be able to subtract numbers with different numbers of digits;* ✓ *using this method, children should also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds;* ✓ *know that decimal points should line up under each other.* £8.95 − £4.38 = 8 + 0.9 + 0.05 − 4 + 0.3 + 0.08 = 8 + 0.8 + 0.15 − 4 + 0.3 + 0.08 *(adjust from T to U)* = 4 + 0.5 + 0.07 = £4.57 leading to 1 8.95 − 4.38	Children will continue to use arrays where appropriate leading into the grid method of multiplication. 6 × 14: 10 and 4; 6 × 10 = 60, 6 × 4 = 24 (6 × 10) + (6 × 4) 60 + 24 84 ✓ **Grid method** **TU x U** (Short multiplication – multiplication by a single digit) 23 x 8 Children will approximate first 23 x 8 is approximately 25 x 8 = 200 × 20 3 8 160 24 160 + 24 184	Children will develop their use of repeated subtraction to be able to subtract multiples of the divisor. Initially, these should be multiples of 10s, 5s, 2s and 1s – numbers with which the children are more familiar. 72 ÷ 5 -2 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 -5 0 2 7 12 17 22 27 32 37 42 47 52 57 62 67 72 Moving onto: -50 r2 -5 -5 -5 -5 -5 0 2 7 12 17 22 72 Then onto the vertical method: **Short division TU ÷ U** 72 ÷ 3 3) 72 - 30 (10×) 42 - 30 (10×) 12 - 6 (2×) 6 - 6 (2×) 0 Answer : 24 Leading to subtraction of other multiples. 96 ÷ 6 16 6) 96 - 60 (10×) 36 - 36 (6×) 0 Answer : 16 Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2. Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division.

Mathematical Calculations Progression

	Addition	Subtraction	Multiplication	Division
Y5	Children should extend the carrying method to numbers with at least four digits. 587 + 475 = 1062 (1 1) 3587 + 675 = 4262 (1 1 1) *Using similar methods, children will:* ✓ *add several numbers with different numbers of digits;* ✓ *begin to add two or more decimal fractions with up to three digits and the same number of decimal places;* ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m – 280 cm.*	**Partitioning and decomposition** Step 1: 754 = 700 + 50 + 4; − 286 = − 200 + 80 + 6 Step 2: 700 + 40 + 14 (*adjust from T to U*); − 200 + 80 + 6 Step 3: 600 + 140 + 14 (*adjust from H to T*); − 200 + 80 + 6; 400 + 60 + 8 = 468 This would be recorded by the children as 600 140 / 7 5 14 / − 2 8 6 / 4 6 8 **Decomposition** 614 1 / 7 ~~5~~ ~~4~~ / − 2 8 6 / 4 6 8 *Children should:* ✓ *be able to subtract numbers with different numbers of digits;* ✓ *begin to find the difference between two decimal fractions with up to three digits and the same number of decimal places;* *know that decimal points should line up under each other* Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used. 1209 – 388 = 821 	**Grid method** **HTU x U** (Short multiplication – multiplication by a single digit) 346 x 9 Children will approximate first 346 x 9 is approximately 350 x 10 = 3500 × 300 40 6 9 2700 360 54 2700 + 360 + 54 = 3114 (1 1) **TU x TU** (Long multiplication – multiplication by more than a single digit) 72 x 38 Children will approximate first 72 x 38 is approximately 70 x 40 = 2800 × 70 2 30 2100 60 8 560 16 2100 + 560 + 60 + 16 = 2736 (1) *Using similar methods, they will be able to multiply decimals with one decimal place by a single digit number, approximating first. They should know that the decimal points line up under each other.* e.g. 4.9 x 3 Children will approximate first 4.9 x 3 is approximately 5 x 3 = 15 × 4 0.9 3 12 2.7 12 + 2.7 = 14.7	Children will continue to use written methods to solve short division TU ÷ U. Children can start to subtract larger multiples of the divisor, e.g. 30x **Short division HTU ÷ U** 196 ÷ 6  Any remainders should be shown as integers, i.e. 14 remainder 2 or 14 r 2. Children need to be able to decide what to do after division and round up or down accordingly. They should make sensible decisions about rounding up or down after division.

Mathematical Calculations Progression

	Addition	Subtraction	Multiplication	Division
Y6	Children should extend the carrying method to number with any number of digits. 7648 + 1486 = 9134 (1 1 1) 6584 + 5848 = 12432 (1 1 1) 42 + 6432 + 786 + 3 + 4681 = 11944 (1 2 1) *Using similar methods, children will* ✓ *add several numbers with different numbers of digits;* ✓ *begin to add two or more decimal fractions with up to four digits and either one or two decimal places;* ✓ *know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 401.2 + 26.85 + 0.71.*	**Decomposition** 5 13 1 6467 - 2684 3783 *Children should:* ✓ *be able to subtract numbers with different numbers of digits;* ✓ *be able to subtract two or more decimal fractions with up to three digits and either one or two decimal places;* ✓ *know that decimal points should line up under each other.* Where the numbers are involved in the calculation are close together or near to multiples of 10, 100 etc counting on using a number line should be used. 3002 – 1997 = 1005  	**ThHTU x U** (Short multiplication – multiplication by a single digit) 4346 x 8 Children will approximate first 4346 x 8 is approximately 4346 x 10 = 43460 x 4000 300 40 6 8 32000 2400 320 48 32000 + 2400 + 320 + 48 = 34768 **HTU x TU** (Long multiplication – multiplication by more than a single digit) 372 x 24 Children will approximate first 372 x 24 is approximately 400 x 25 = 10000 x 300 70 2 20 6000 1400 40 4 1200 280 8 6000 + 1400 + 1200 + 280 + 40 + 8 = 8928 (1) *Using similar methods, they will be able to multiply decimals with up to two decimal places by a single digit number and then two digit numbers, approximating first. They should know that the decimal points line up under each other.* *For example:* 4.92 x 3 Children will approximate first 4.92 x 3 is approximately 5 x 3 = 15 x 4 0.9 0.02 3 12 2.7 0.06 12 + 0.7 + 0.06 = 12.76	Children will continue to use written methods to solve short division TU ÷ U and HTU ÷ U. **Long division HTU ÷ TU** 972 ÷ 36 27 36) 972 - 720 (20x) 252 - 252 (7x) 0 Answer: 27 Any remainders should be shown as fractions, i.e. if the children were dividing 32 by 10, the answer should be shown as $3\ ^{2}/_{10}$ which could then be written as $3\ ^{1}/_{5}$ in it's lowest terms. Extend to decimals with up to two decimal places. Children should know that decimal points line up under each other. 87.5 ÷ 7 12.5 7) 87.5 - 70.0 (10x) 17.5 - 14.0 (2x) 3.5 - 3.5 (0.5x) 0 Answer : 12.5

By the end of year 6, children will have a range of calculation methods, mental and written. Selection will depend upon the numbers involved.
Children should not be made to go onto the next stage if:
they are not ready.
they are not confident.
Children should be encouraged to approximate their answers before calculating.
Children should be encouraged to consider if a mental calculation would be appropriate before using written methods.